Chapter 11

Capital investment appraisal

Real world case 11.1

This case study shows a typical situation in which management accounting can be helpful. It also shows how the descriptions used by an organisation may change over time. Read the case study now but only attempt the discussion points after you have finished studying the chapter.

The following report indicates that Network Rail was focused on investment appraisal procedures in 2004 at the start of a five-year regulatory period.

John Armitt, chief executive of Network Rail, the 'not for dividend' successor to Railtrack as owner of the UK rail infrastructure, does not expect an ongoing review of the industry's performance payment system to produce any significant changes.

'The first half of 2004/5 marked the start of the new five-year regulatory period, which has put the company on a sound financial footing. Our new investment appraisal procedures are ensuring greater efficiency and maximising the effectiveness of every pound in the ground', said Armitt.

Source: Network Rail's Armitt sees no change to performance payment system AFX Europe (Focus); 26 November 2004.

The Annual Report 2009 shows a very different use of wording at the end of the five-year period. There is a strong focus on operational targets such as completion of line renewal, and on the magnitude of the investment programme, but no mention of 'investment appraisal' or 'capital expenditure'.

'Our investment programme to enhance and modernise the network is the most ambitious it has ever been. We are delivering a 21st century railway for our customers and society at large.'

. . .

'It is important to note that the measurement of efficiency improvement against these targets is not, and will never be, a straightforward exercise. The determination did not define baseline volumes of activity or unit costs against which changes could be measured, and there is limited information on the unit costs of activities in 2003/04 to provide benchmarks. The assessment of efficiency improvement over CP3 [the five-year period from 2004] set out here must be treated with caution as firm conclusions on efficiency rely on assessments of sustainability that, due to the long lifespan of railway assets, can only be assessed over a longer period of time.'

Source: Annual Report 2009, http://www.networkrail.co.uk, inside cover page and p. 167.

Discussion points

1 What questions would you ask in appraising investment in a railway track or a railway station?

2 How easy is it to carry out an evaluation of the success of an investment project?

Contents

Learning outcomes

After reading this chapter you should be able to:

- Explain the purpose of capital appraisal and the role of the management accountant.
- Explain the payback method and calculate the payback period.
- Explain and calculate the accounting rate of return.
- Explain and calculate the net present value of a project.
- Explain and calculate the internal rate of return of a project.
- Describe and discuss examples of research into the use of different forms of investment appraisal.

11.1 Capital project planning and decisions

The word 'capital' can have more than one meaning in accounting. In financial reporting in particular it is used to denote the finance provided to the business by owners and long-term lenders. Economists use the term 'capital' to refer to the fixed assets and working capital of a business which are purchased with the money provided by the owners and lenders. This chapter uses the term 'capital' in a manner similar to that used by the economists.

When the managers of a business make plans for the long term they have to decide whether, and how much, to invest in fixed assets and working capital to maintain or increase the productive capacity of the business. They will usually be faced with choices of projects available, each requiring a different type of investment, and with only a limited amount of finance available. They have to ask themselves a number of questions, including:

1 How many of the proposed projects are worth undertaking?
2 How much finance, in total, should we commit to new projects?
3 Where should the finance be obtained?
4 After the event, was the investment in the proposed project successful?

These questions cross an academic spectrum of study which begins in management accounting and ends in finance. The first and fourth of these questions are normally dealt with in management accounting textbooks, while the second and third form the focus of finance textbooks. Some books in either discipline will attempt to deal with all the questions. This chapter focuses on the first and fourth questions. It explains techniques that can be applied to evaluate ('appraise') an investment project in order to decide whether it is worthwhile to start the project.

Definition

Capital investment appraisal is the application of a set of methods of quantitative analysis which give guidance to managers in making decisions as to how best to invest long-term funds.

11.1.1 The role of the management accountant in capital investment appraisal

The management accountant's role was set out in Chapter 1 as directing attention, keeping the score and solving problems. In **capital investment appraisal** it is the role of directing attention which is important. Information about proposed capital projects must be presented in a way which will direct management's attention towards the significant information for decision-making purposes. There will most probably be problems to solve in terms of gathering and presenting the information. After the project is implemented there will be a score-keeping aspect in terms of comparing the actual outcome with the plans and expectations.

This chapter concentrates on the techniques of presenting information so as to direct attention to the significant aspects of the capital project for decision-making purposes. It concludes with an explanation of how a project may be evaluated after it is completed (called a post-completion audit).

11.1.2 The assumptions adopted

Certainty of cash flows

This chapter makes the assumption that all future cash inflows and outflows of a long-term project may be predicted with certainty. Making an assumption of certainty may

seem a rather unrealistic starting point, but it is necessary to do so in order to analyse the principles of capital investment appraisal without having too many real-world complications crowding in.

No taxes, no inflation

This chapter also assumes that there are no taxes and no inflation to cause prices to increase over the life of the project.

Timing of cash flows

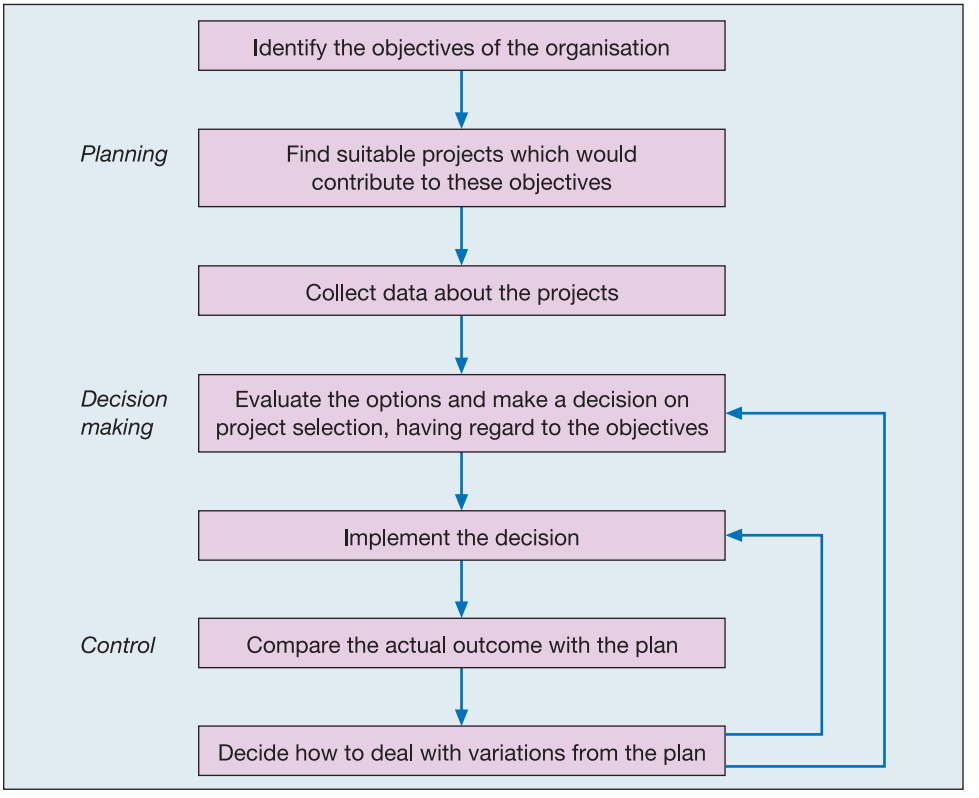
For the calculations described in this chapter, simplifying assumptions are made about the timing of cash flows. In the payback method and the accounting rate of return the cash flows are assumed to be spread evenly throughout the accounting period. In the net present value method the cash flows are assumed to arise at one point in time, on the final day of the accounting period. These simplifying assumptions are necessary to allow simple models to be created for calculation. The unevenness of cash flows in practice is another real-world complication.

11.1.3 Making a decision on a capital investment

Chapter 1 contains a description of the processes of planning and control which are necessary for a systematic approach to making an investment decision in locating a new retail outlet. In general terms, that process is as shown in Figure 11.1.

To be successful the business must first of all discover projects which have the potential for success. All the management accounting in the world will not create a successful project. The successful entrepreneur is the person who has the flair and

Figure 11.1
Planning and control for a capital investment decision



imagination to identify projects and to see how they might successfully operate. The role of management accounting, through the capital investment appraisal process, is to ensure that the excitement of creating new investment opportunities does not cause management to lose sight of the need to meet the organisation's objectives.

11.1.4 Selecting acceptable projects

Suppose there has been a meeting of the board of directors of a company at which the managing director has said: '*We want to ensure that any cash we invest in a project comes back as soon as possible in the form of cash flows which give us a profit overall and provide the cash to reinvest in the next project.*'

A second director has replied by saying: '*It's fine for you to talk about cash flows but the outside world judges our success by our profit record. I would like to see us choosing projects which maximise the return on assets invested in the project.*'

A third member of the board has joined in with: '*I agree with the cash flow perspective, but I want to be sure that, at the minimum, we cover the interest charges we have to pay on the money we borrow to finance the project. Ideally, there should be cash flows generated which exceed the cost of borrowing, so that we have surplus funds to use for investment in further projects or for increasing dividends to our shareholders.*'

Reading carefully what each has said, it is apparent that there are similarities and differences in the targets they would like to set. They are all looking to the cash flows that will be generated from the project, but the first director is emphasising the speed of collecting cash flows, while the second director wants to convert cash flows to profit by deducting depreciation, and the third director is more concerned about the amount of cash flows in total and whether they provide a surplus after covering all costs.

Management accounting can provide information for capital investment appraisal purposes which would satisfy the criteria set by any one of the three directors, but there would remain the question as to which of the three directors is using the best approach so far as the business is concerned. Four methods of capital investment appraisal will now be explained. These are: the payback method, the accounting rate of return, the net present value method and the internal rate of return method. Each management accounting technique will be described in turn and the advantages and disadvantages of each will be discussed.

Activity 11.1

Decide now which of the three directors you think has the most desirable approach and why you think that way. Then monitor the development of your views as you read the chapter.

11.2 Payback method

11.2.1 Method of calculation

The first director wanted cash invested in a project to come back as quickly as possible in the form of cash flows. To test whether this objective has been met by a capital project, the payback method of project appraisal is used. It provides a calculation of the length of time required for the stream of cash inflows from a project to equal the original cash outlay. The most desirable project, under the payback method, is the one which pays back the cash outlay in the shortest time. Data are set out in Exhibit 11.1 which will be used to illustrate all the capital investment appraisal methods explained

in this chapter. An illustration of the payback calculation is provided in Table 11.1, and from this table of calculations it may be seen that project A offers the *shortest* **payback period**. Thus, if the most important measure of success in investment is the recovery of the cash investment, then Project A is the preferred choice. Project C is next in rank and Project B is the least attractive.

Definitions

The **payback method** of project appraisal calculates the length of time required for the stream of cash inflows from a project to equal the original cash outlay.

The **payback period** is the length of time required for a stream of net cash inflows from a project to equal the original cash outlay.

Exhibit 11.1
Data for illustration of methods of capital investment appraisal

Data
A haulage company has three potential projects planned. Each will require investment in two refrigerated vehicles at a total cost of £120,000. Each vehicle has a three-year life. The three projects are:

A Lease the vehicles to a meat-processing factory which will take the risks of finding loads to transport and will bear all driver costs for a three-year period. Expected net cash inflows, after deducting all expected cash outflows, are £60,000 per annum.

B Enter into a fixed-price contract for three years to carry frozen foods from processing plants in the UK to markets in Continental Europe, returning with empty vehicles. This will require employing drivers on permanent contracts. Expected cash inflows, after deducting all expected cash outflows, are £45,000 per annum.

C Employ a contracts manager to find loads for outward and return journeys but avoid any contract for longer than a six-month period so as to have the freedom to take up opportunities as they arise. Drivers will be hired on short-term contracts of three months. Expected cash inflows, after deducting all expected cash outflows, are £40,000 in Year 1, £70,000 in Year 2 and £80,000 in Year 3.

Table 11.1
Calculations for payback method

Cash flows	*Project A*	*Project B*	*Project C*
	£	£	£
Outlay	120,000	120,000	120,000
Cash inflows, after deducting all outflows of the year			
Year 1	60,000	45,000	40,000
Year 2	60,000	45,000	70,000
Year 3	60,000	45,000	80,000
Payback period	2 years	2.67 years	2.125 years
Workings	$60 + 60 = 120$	$45 + 45 + \frac{30}{45}$	$40 + 70 + \frac{10}{80}$

11.2.2 Impact of uncertainty in real life

This calculation assumes certainty about the cash flows predicted for each project. It also assumes that cash flows are spread evenly throughout the accounting period. Hopefully, as you were reading the conditions of the three different contracts set out in Exhibit 11.1, you had some thoughts about the relative commercial risk of each project and the risks attached to the cash flows. In this chapter we do not make allowance for the relative risks of each project, because we make an assumption of certainty of predicted cash flows but, in real life, Project C would be regarded commercially as the high-risk option, while projects A and B provide greater certainty through having contracts in place for the three-year period. Of these two, project B looks the less attractive but leaves opportunities for casual earnings if loads can be found for the return journey.

11.2.3 Usefulness and limitations of the payback approach

The payback method of capital investment appraisal is widely used in practice, possibly because it is relatively painless in its arithmetic. Furthermore, there is a reflection of commercial realism in concentrating on projects which give early returns of cash flow. That may be important to organisations which face cash flow constraints. It may also be seen as a cautious approach to take where product markets are uncertain and it is difficult to predict the longer-term cash flows expected from a product.

One major limitation of using the payback method of capital investment appraisal as described here is that it ignores the fact that investing funds in a long-term project has a cost in terms of the interest charged on borrowed funds (or interest forgone when money is tied up in fixed assets). Economists refer to this interest cost as the **time value of money**. This is the name given to the idea that £1 invested today will grow with interest rates over time (e.g. £1 becomes £1.10 in one year's time at a rate of 10 per cent).

Definition

The **time value of money** is the name given to the idea that £1 invested today will grow with interest rates over time (e.g. £1 becomes £1.10 in one year's time at a rate of 10 per cent).

The cash flows earned from a project should repay the capital sum invested, but they should also be sufficient to provide a reward to investors which equals the interest cost of capital.

A second major limitation is that, in concentrating on the speed of recovery of cash flows, the method ignores any cash flows arising after the payback date. A project which would make a long-term contribution to the overall cash flows of the business could be sacrificed for short-term benefits in a project with a limited time horizon.

Activity 11.2

Check that you understand fully the calculation of the payback period and its interpretation. Check also that you can explain the meaning and usefulness of the payback period as a means of evaluating the suitability of a project.

11.3 Accounting rate of return

11.3.1 Method of calculation

The **accounting rate of return** differs from the payback method in using *accounting profits* rather than cash flows. The calculation of profits includes depreciation, which is an accounting allocation but has no cash flow effect. The attraction of using profit in a

method of capital investment appraisal is that it links long-term decision making to profit as the conventional measure of success in business.

Definitions

The **accounting rate of return** is calculated by taking the average annual profits expected from a project as a percentage of the capital invested.

Average annual profit is calculated as average annual cash flow minus annual depreciation.

Some textbooks recommend as denominator the initial amount of capital invested while others suggest the use of the average capital invested. Calculation of the average involves making some arbitrary assumptions about the way capital is used up over the project. A simple pattern is to assume it is used up evenly. Suppose a project requires £1,000 invested at the start, there will be nothing left at the end and the capital is used up equally each year. Then the average investment is £500 (which is the average of £1,000 at the start and £nil at the end). This textbook will use the initial investment for illustrative purposes, but you should be aware that different definitions will be used in practice and it is important to know how any return on capital has been defined.

The data in Exhibit 11.1 may be used to illustrate the accounting rate of return as a method of capital investment appraisal. A straight-line method of depreciation is applied, assuming a zero residual value, so that depreciation of £40,000 per annum (calculated as £120,000/3) is deducted from cash flows. The resulting profits and accounting rate of return are shown in Table 11.2.

Table 11.2 shows that Project C has the highest rate of return, Project A is next in rank and Project B has the lowest rate of return. The accounting rate of return gives a ranking of the three projects different from that given by the payback method. Project B remains the least attractive but the positions of Projects A and C are reversed. C creates more cash flow in total, but the cash flows of A arise earlier than those of C.

Table 11.2
Calculations for the accounting rate of return

Cash flows	*Project A*	*Project B*	*Project C*
	£	£	£
Outlay (a)	120,000	120,000	120,000
Profits, after deducting depreciation from cash flows			
Year 1	20,000	5,000	nil
Year 2	20,000	5,000	30,000
Year 3	20,000	5,000	40,000
Average annual profit (b)	20,000	5,000	23,000
Accounting rate of return (b × 100/a)	16.7%	4.2%	19.2%

11.3.2 Usefulness and limitations of accounting rate of return

The accounting rate of return is regarded as a useful measure of the likely success of a project because it is based on the familiar accounting measure of profit. It is also regarded as useful because it takes into the calculation all the profits expected over the

project life (in contrast to the payback method which ignores all cash flows beyond the payback date). It assumes an even spread of cash flows throughout the accounting period.

A major defect of the accounting rate of return is that it ignores the time value of money. The time value of money means that there is greater value in a cash flow of £1 promised next year than in a cash flow of £1 promised in a later year. The accounting rate of return makes no distinction between two projects of the same average profit, one of which gives most of its profits at an early stage and the other of which gives most of its profits at a later stage.

A less serious defect, but nevertheless a limitation, is that the accounting rate of return depends on profit which, in turn, includes a subjective accounting estimate of depreciation. That may not matter too much in an example of the type illustrated in Table 11.2, where average profits are used and straight-line depreciation is applied across all projects, but there could be situations where different depreciation policies could distort a decision based on the accounting rate of return.

Activity 11.3

Before proceeding further, make sure that you understand fully the calculation and usefulness of the accounting rate of return. Check also that you understand the limitations of relying on the accounting rate of return when evaluating a project.

Real world case 11.2

This extract from the annual report of Punch Taverns refers to 'return on investment' (ROI) as an alternative description of the accounting rate of return.

A further £48.7m was invested in the acquisition of 80 individual pubs during the year, together with investment of £46.2m on existing pubs within the estate and £5.0m on infrastructure. We continue to see excellent returns on our pub investments and good opportunities to develop our estate further. Of the £46.2m investment, £34.9m was spent on 580 profit enhancing projects (including 74 from the Pubmaster estate), generating a first-year pre-tax ROI of 29 per cent.

Source: Punch Taverns plc, Annual report 2004, p. 16. www.punchtaverns.com.

Discussion points

1 How will investors form a view on the accounting rate of return (ROI)?

2 How does the company reassure investors about the value of the investments in a non-quantified way?

11.4 Net present value method

The net present value (NPV) method of capital investment appraisal is a technique which seeks to remedy some of the defects of payback and the accounting rate of return. In particular, it takes into account all cash flows over the life of the project and

makes allowance for the time value of money. Before the net present value method can be explained further, it is necessary to say more about the time value of money.

11.4.1 Time value of money

If £100 is invested at 10 per cent per annum, then it will grow to £110 by the end of the year. If the £100 is spent on an item of business machinery, then the interest is lost. So the act of investing leads to a lost opportunity of earning investment. The idea of applying calculations of the time value of money is a way of recognising the reward needed from a project to compensate for the lost opportunity.

Suppose now that you have been given a written promise of £100 to be received in one year's time. Interest rates are 10 per cent. You do not want to wait one year to receive cash and would like the money now. What is the price for which you could sell that promise? Most students see the answer as £90.91 intuitively, but they do not all see immediately how they arrived at that answer. (It might be useful for you to think out your own approach before you read the next few paragraphs. It is much easier to work something out for yourself than to try remembering formulae which you will forget in a crisis.)

The intuitive answer is that £90.91 is the amount which, invested now at 10 per cent, would grow to £100 in one year's time. Provided the promise is a good one, there would be no problem in selling the £100 promise for £90.91 now. Both the buyer and the seller would be equally satisfied that the price reflected the time value of money.

Now make it a little harder. Suppose the promise of £100 was for payment in *two* years' time. What is the price for which you could sell that promise now? The answer is £82.64 because that would grow at 10 per cent to £90.91 at the end of one year and to £100 at the end of two years.

The calculation of the value of the promise today can be conveniently represented in mathematical notation as follows:

Definition

The **present value** of a sum of £1 receivable at the end of n years equals:

$$\frac{1}{(1+r)^n}$$

Where r represents the annual rate of interest, expressed in decimal form, and n represents the time period when the cash flow will be received.

The process of calculating present value is called *discounting.* The interest rate used is called the *discount rate.*

Using this **discounting** calculation to illustrate the two calculations already carried out intuitively, the present value of a sum of £100, due one year hence, when the **discount rate** (interest rate) is 10 per cent, is calculated as:

$$\frac{£100}{(1+0.1)^1} = £90.91$$

The **present value** of a sum of £100, due two years' hence, when the interest rate is 10 per cent, is calculated as:

$$\frac{£100}{(1+0.1)^2} = £82.64$$

The calculation using this formula is no problem if a financial calculator or a spreadsheet package is available, but can be tedious if resources are limited to a basic pocket calculator. In such circumstances, some people prefer to use tables of discount

factors which give the present value of £1 for every possible rate of interest and every possible time period ahead. A full table of discount factors is set out in the appendix at the end of this chapter (p. 287). As an example from that supplement, the column for the discount rate of 10 per cent has the following discount factors:

At end of period	*Present value of £1*
1	0.909
2	0.826
3	0.751

Using the tables, for the discount rate of 10 per cent, it may be calculated that the present value of £100 receivable at the end of Year 1 is £100 × 0.909 = £90.90, while the present value of £100 receivable at the end of Year 2 is £100 × 0.826 = £82.60. (There is a difference in the second place of decimals when this answer is compared with the result of using the formula. The difference is due to rounding in the discount tables.)

In these calculations it is assumed that cash flows all arise on the final day of the relevant accounting period.

Now that you are familiar with the calculation of the present value of a promised future cash flow, the explanation of the net present value method of capital investment appraisal may be given.

Activity 11.4

Use your calculator to check the discount factors for the present value of £1 at the end of one year, two years and three years for a discount rate of 10%. Write a parallel table for 8 per cent and 12 per cent. Show that the discount factor decreases as the discount rate increases.

11.4.2 The net present value decision rule

The net present value (NPV) method of capital investment appraisal is based on the view that a project will be regarded as successful if the present value of all expected inward cash flows is greater than, or equal to, the capital invested at the outset. It is called *net* present value because, in calculation, the capital invested is deducted from the present value of the future cash flows. (Use of the word 'net' always means that one item is being deducted from another.) If the present value of the expected cash flows is greater than the capital invested, then the net present value will be positive. If the present value of the expected cash flows is less than the capital invested, then the net present value will be negative. A positive net present value indicates that the project should be accepted, while a negative net present value indicates that it should be rejected.

Definitions

The **net present value (NPV)** of a project is equal to the present value of the cash inflows minus the present value of the cash outflows, all discounted at the cost of capital.

Cash flows are calculated as profit before deducting depreciation and amortisation.

The NPV decision rule is as follows:

Definition

Decision rule: NPV

- Where the net present value of the project is *positive,* accept the project.
- Where the net present value of the project is *negative*, reject the project.
- Where the net present value of the project is zero, the project is acceptable in meeting the cost of capital, but gives no surplus to its owners.

If an organisation seeks to maximise the wealth of its owners, then it should accept any project which has a positive net present value. If finance markets are working efficiently, funds will always be available to finance projects which meet or exceed their cost of capital.

11.4.3 The cost of capital

The rate of interest used in the calculation of net present value is called the discount rate. It is based on the cost to the business of raising new finance. This is called the **cost of capital**. If the project is to be financed only by borrowing from banks then the cost of capital is the rate of interest that a bank would charge for a new loan. If the project is to be financed only by issuing new share capital, then the cost of capital is the dividend yield required by investors. If the project is to be financed by cash that has been saved within the business, then the shareholders have allowed this saving rather than take a dividend, so the cost of capital is the opportunity cost reflected in the dividend yield.

When the business finances projects by a mixture of sources of finance, the cost of capital is a mixture of the related costs. It is calculated by a weighted average of the interest rate on loans and the dividend yield on share capital. The weights used are based on the relative amounts of loan finance and equity finance used by the company. If you study corporate finance you will learn more about estimating the weighted average cost of capital. Investors may expect a higher rate of return on their investment for a project of higher risk. The cost of capital may therefore depend on the risks associated with a project. For any exercise in this textbook you will be informed of the discount rate to be used.

11.4.4 Residual value

At the end of a project's life there may be cash flows that can be collected from sale of equipment or recovery of cash invested in inventories and debtors. Any cash flows from **residual value** should be included in the projected cash flows and discounted from the end of the project.

11.4.5 Illustration

The illustration in Table 11.3 sets out the data for Project A taken from Exhibit 11.1. Table 11.4 sets out the net present value calculation, assuming a discount rate of 10 per cent. Based on the net present value rule Project A will be accepted as it gives a positive net present value.

Table 11.3
Data for net present value illustration

Cash flows	*Project A* £
Outlay	120,000
Cash inflows, after deducting all outflows of the year:	
Year 1	60,000
Year 2	60,000
Year 3	60,000

Table 11.4
Calculation of net present value: Project A

Using the formula approach the net present value is calculated as:

$$\frac{£60{,}000}{(1.10)} + \frac{£60{,}000}{(1.10)^2} + \frac{£60{,}000}{(1.10)^3} - £120{,}000$$

$$= £54{,}550 + £49{,}590 + £45{,}080 - £120{,}000 = £29{,}200$$

Using the discount tables the net present value is calculated as:

End of year	*Cash flow* £	*Discount factor*	*Present value* £
0 Initial outlay	(120,000)	1.000	(120,000)
1	60,000	0.909	54,540
2	60,000	0.826	49,560
3	60,000	0.751	45,060
Present value of cash flows			149,160
Less initial outlay			(120,000)
Net present value			29,160

Rounding errors

The answer obtained from the discount tables (£29,160) differs marginally from that obtained from the formula (£29,220), because the discount factors are rounded to three decimal places. In many cases, such differences are marginal to the overall calculation and you should not worry about them. If, in any particular case, the rounding errors are likely to have an impact, then the formula should be used rather than the tables of discount factors. In real life it is questionable whether any decision should be based on fine-tuning of rounding errors. The conclusion should be clear from the overall magnitudes being calculated and should not be dependent on differences of very small magnitude.

Activity 11.5

If you have access to a spreadsheet package, find out whether it has a net present value (NPV) function. If so, use the data in Table 11.3 to satisfy yourself that the spreadsheet produces answers similar to those derived here.

Cash flow patterns assumed by the net present value calculation

It is worth pausing to analyse the cash flow patterns which are assumed by the net present value calculation. This analysis helps in understanding when it is safe to use the net present value approach to capital investment appraisal and when it should be applied with caution.

Assume the investor who has provided the capital of £120,000 requires 10 per cent interest at the end of each year, to be paid out of the cash flows. Assume that any surplus cash flows are retained in the business and reinvested at 10 per cent. The accumulation of cash generated by the project is shown in Table 11.5. The cash balance at the end of Year 3 is £159,000, out of which the original capital of £120,000 is repaid, leaving an actual surplus of £39,000. That surplus arising at the end of Year 3 has a present value of £29,000 (£39,000 × 0.751) which is the answer derived earlier by the net present value calculation (allowing for rounding differences).

Table 11.5 is provided here to illustrate one of the assumptions of the net present value calculation which requires some thought. It assumes that surplus cash generated during the project can be invested at the cost of capital. Whether or not that is the case for a particular project is more an issue for study in the area of finance, but in real life it is rare that the interest earned on deposited funds is as high as that paid on borrowings. What is possible in many situations is that the surplus cash is used to start further projects in the business and those new projects are also successful in creating positive net present values of cash flows at the organisation's cost of capital.

Table 11.5
Accumulation of cash during a project

Year	*Balance of cash at start of year* (1) £000s	+	*Interest earned on balance invested* (2) £000s	+	*Cash flow* (3) £000s	–	*Interest paid* (4) £000s	=	*Balance of cash at end of year* (1 + 2 + 3 – 4) £000s
1	nil	+	–	+	60	–	12	=	48
2	48	+	5	+	60	–	12	=	101
3	101	+	10	+	60	–	12	=	159

Projects B and C

Now consider Projects B and C. The net present value of each project is calculated in Tables 11.6 and 11.7.

Table 11.6
Calculation of net present value: Project B

Using the discount tables the net present value is calculated as follows:			
End of year	*Cash flow* £	*Discount factor*	*Present value* £
1	45,000	0.909	40,905
2	45,000	0.826	37,170
3	45,000	0.751	33,795
			111,870
Less initial outlay			(120,000)
Net present value			(8,130)

Project C has the highest net present value, followed by Project A. Both would be acceptable because both have a positive net present value. Project B would be rejected because it gives a negative net present value.

In real life, obtaining finance may be difficult because of temporary imbalance in the capital markets or because the supply of capital within the organisation is constrained. If the organisation is in the public sector it may be subject to a cash limit of capital expenditure. If it is in the private sector and is a subsidiary or a division within a group, it may be restricted by the group's plans for total fund-raising by the group. Such practical problems are sometimes referred to as **capital rationing** and will lead to

Table 11.7
Calculation of net present value: Project C

Using the discount tables the net present value is calculated as follows:			
End of year	*Cash flow* £	*Discount factor*	*Present value* £
1	40,000	0.909	36,360
2	70,000	0.826	57,820
3	80,000	0.751	60,080
			154,260
Less initial outlay			(120,000)
Net present value			34,260

organisations devising decision rules for ranking projects. These ranking decisions will not be explored in detail here but it is important to note that any project which is rejected, when it has a positive net present value, will be a loss to the potential wealth of the owners of the business.

11.4.6 Impact of uncertainty

In real life it is unlikely that cash flows for each accounting period can be predicted with certainty. It is also unlikely that the cost of capital can be estimated precisely. One method of dealing with this kind of uncertainty is to carry out net present value calculations for a range of scenarios, using the 'best, worst, most likely' approach shown in Chapter 10.

11.5 Internal rate of return

Net present value is only one method in capital investment appraisal which takes into account the time value of money. The decision rule is based on the absolute amount of the net present value of the surplus generated by the project. There is some evidence from research into the practical use of capital investment appraisal techniques that decision makers feel more comfortable with a percentage rather than an absolute amount. (The reason is not so clear, but could be linked to the historical reliance on the accounting rate of return as a percentage.)

The **internal rate of return (IRR)** is another method in capital investment appraisal which uses the time value of money but results in an answer expressed in percentage form. It is a discount rate which leads to a net present value of zero, where the present value of the cash inflows exactly equals the cash outflows.

Definition

The **internal rate of return (IRR)** is the discount rate at which the present value of the cash flows generated by the project is equal to the present value of the capital invested, so that the net present value of the project is zero.

11.5.1 Method of calculation

The calculation of the internal rate of return involves a process of repeated guessing at the **discount rate** until the present value of the cash flows generated is equal to the

capital investment. That guessing may be carried out by computer, asking the computer to try values of the discount factor in the formula. Most spreadsheet computer packages have the facility to perform a calculation of internal rate of return once the initial investment and cash flows have been entered on the spreadsheet.

$$\text{Initial investment} = \frac{C_1}{(1+d)} + \frac{C_2}{(1+d)} + \frac{C_3}{(1+d)} + \cdots + \frac{C_n}{(1+d)^n}$$

That process of repeated guessing is extremely time-consuming if a computer is not used. Even where a computer is used, it needs to be provided with a first guess which is reasonably close. For a manual process of estimation it may be easier to use discount tables, with an aim of arriving at a reasonably close answer, rather than worrying too much about figures beyond the decimal point.

Take, as an illustration, the data on Project A of Exhibit 11.1, repeated in Table 11.3. The starting point for calculating IRR is to find two values of NPV using discount rates lying either side of the IRR. Table 11.8 sets out two such calculations. A first guess of 20 per cent produces a net present value which is positive. The aim is to find the discount rate which gives a zero net present value, so the first guess must have been too low and a higher discount rate of 24 per cent is used for the second guess.

Table 11.8
Calculation of net present value at 20 per cent and at 24 per cent

	Cash flows £	*Discount rate 20%* £		*Discount rate 24%* £	
End of Year 1	60,000	0.833	49,980	0.806	48,360
End of Year 2	60,000	0.694	41,640	0.650	39,000
End of Year 3	60,000	0.579	34,740	0.524	31,440
			126,360		118,800
Outlay			(120,000)		(120,000)
Net present value			6,360		(1,200)

The second guess was a fortunate one because the net present value changed from being positive at 20 per cent to being negative at 24 per cent. That means that the net present value of zero must be found at a discount rate between these two rates. If the second guess had failed to give a negative net present value, a further guess would have been required.

The actual discount rate which gives a zero net present value may now be found by assuming a linear interval between 20 per cent and 24 per cent. (The interval is not exactly linear but may be taken as approximately so over a narrow difference in rates.)

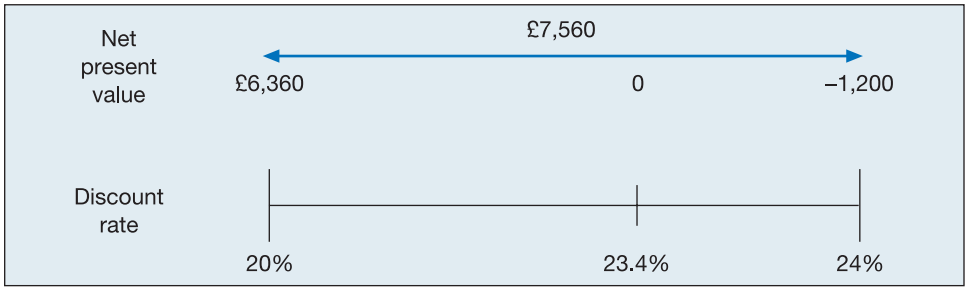
The difference between the two net present values is £6,360 – (–£1,200), that is £7,560. The difference between the two discount rates is four per cent and therefore, using simple proportion calculations, the net present value of zero lies at:

$$20\% + \left(\frac{6{,}360}{7{,}560} \times 4\right) = 23.365\%$$

Figure 11.2 sets out the linear relationship which is assumed in the calculation. The process of estimation shown there is called *interpolation*. In words, the formula used in this calculation is:

$$\text{Lower of the pair of discount rates} + \left(\frac{\text{NPV at lower rate}}{\text{Difference between the NPVs}} \times \text{Difference in rates}\right)$$

Figure 11.2
Locating the internal rate of return between two discount rates of known net present value



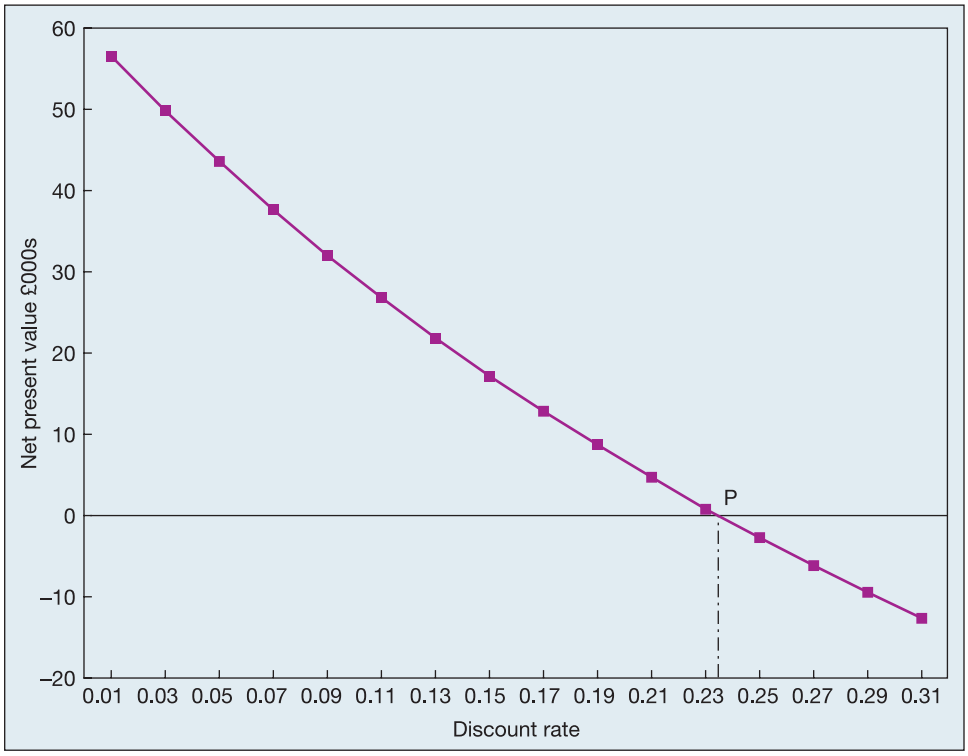
The internal rate of return answer, as produced by a computer package, is 23.375 per cent. The use of a simple proportion calculation appears to provide a good approximation.

Activity 11.6

If you have access to a computer spreadsheet package which has an internal rate of return function, test the data used in the chapter. It will ask you for a first guess and will then proceed to repeat the calculation of IRR until it arrives at a net present value of zero.

It is also possible to plot a graph of net present value against discount rate, as shown in Figure 11.3. The IRR is the discount rate at which the graph crosses the horizontal line representing zero net present value. That point is designated with a letter P in the graph and is shown to be around 23.4 per cent by a vertical dotted line from P to the horizontal axis.

Figure 11.3
Graph of net present value against discount rate showing internal rate of return



11.5.2 The internal rate of return decision rule

The decision rule is that a project is acceptable where the internal rate of return (IRR) is greater than the cost of capital. Under those conditions the net present value of the project will be positive. A project is not acceptable where the IRR is less than the cost of capital. Under those conditions the net present value of the project will be negative.

Definition

Decision rule: IRR

- Where the IRR of the project is greater than the cost of capital, accept the project.
- Where the IRR of the project is less than the cost of capital, reject the project.
- Where the IRR of the project equals the cost of capital, the project is acceptable in meeting the required rate of return of those investing in the business, but gives no surplus to its owners.

When the net present value and the internal rate of return criteria are applied to an isolated project, they will lead to the same accept/reject decision because they both use the discounting method of calculation applied to the same cash flows. For an isolated project the use of either technique is a matter of personal preference. Where a choice of competing projects has to be made, the practice may be more complicated. Chapter 12 explains the evaluation of projects that are competing for scarce resources.

Real world case 11.3

Senior Management Appointment

ntl Incorporated has appointed Jacques Kerrest as its Chief Financial Officer. As CFO, Jacques is responsible for all of ntl's financial activities including cash and credit management, capital budgeting, financial planning and analysis, corporate finance, tax, financial reporting, SEC and regulatory filings, accounting systems and controls, internal audit, bank relationships, financing and investor relations.

Source: *Business Wire*, 3 November, 2004 'ntl Incorporated's Third Quarter Results Led by Continued Growth in ntl: Home'.

Discussion points

1. One of the tasks listed for the CFO is 'capital budgeting'. What are the other ways in which the CFO is expected to show skills developed under the theme of 'management accounting'?
2. How could you find out more about the capital budgeting methods used by the CFO in this company?

11.6 Which methods are used in practice?

This chapter has now explained the capital budgeting techniques of payback, accounting rate of return, net present value and internal rate of return. The benefits and limitations of each have been discussed in the respective sections. It could be argued

that the proof of the value of each technique lies in the extent to which it is used in practice. There exists a considerable volume of survey research seeking an answer to the question of which methods are most commonly used in practice. The conclusions from each project are not totally unanimous because they depend on the time period covered by the research, the nature of the sample chosen, the country in which the questions are asked and the questions asked. There are themes which may be discerned in the research results, the first of which is that the payback method appears to be the most frequently used technique in the UK but discounted cash flow methods are found more commonly in the USA. It is also found that organisations will use more than one method of capital budgeting. Where discounting methods are used, internal rate of return appears more popular than net present value.

One benefit of using internal rate of return is that large companies with operations in different countries may set different hurdle levels of IRR to reflect the risk inherent in the different countries.

Research investigations are able to collect information of this type. Once the patterns are known, it is interesting to speculate on the motives behind these patterns of choice. Perhaps the payback method is most frequently used because there are many small businesses undertaking lots of small projects. It might not matter that discounting methods are used less frequently provided they are used on the larger projects in larger organisations. This issue has also been tested in research and it has been shown that larger companies do make relatively more use of discounting techniques. Perhaps the payback method, in many cases, shows so clearly that a project is acceptable that it would be a waste of time to carry out lengthy discounting calculations to arrive at the same conclusion. Perhaps those using payback realise that, in some instances, its emphasis on early cash flows is not so different from that of the net present value approach in situations where the later cash flows are relatively low.

11.7 What the researchers have found

11.7.1 Which methods of investment appraisal are used?

Brounen *et al*. (2004) reported a survey of corporate finance in Europe. They asked 313 European chief finance officers (CFOs) about their methods of investment appraisal. The survey was carried out in 2002. Most European respondents selected the payback period as their most frequently used investment appraisal technique. The use of payback was: UK (69.2 per cent of respondents), the Netherlands (64.7 per cent), Germany (50 per cent) and France (50.9 per cent). The use of IRR was strongest in Germany at 56 per cent but NPV was even stronger in Germany at 70 per cent. In the other three countries IRR was more commonly used than NPV.

The researchers expressed some surprise in their conclusions that firms in the UK and the Netherlands were consciously striving to maximise shareholder wealth while firms in France and Germany attached low priority to that corporate goal. It would not surprise someone who knows the relative strength of capital markets in the UK and the Netherlands and the societal focus of French and German companies. Their findings confirmed many previous studies in observing that net present value criteria are more likely to be seen in larger companies.

11.7.2 Caution over using NPV in public-sector evaluation

Cooper and Taylor (2005) reported on an investigation of proposals to the Scottish Parliament which, if implemented, would lead to a considerable expansion of prison privatisation. Both the Scottish Prison Service and the Scottish Executive used what

they claimed to be an independently verified cost saving of £700 million as the major justification for these proposals. The Executive minister who presented the report to the Parliament said 'In order to compare the costs of the options, they have been assessed on the standard Treasury-approved net present value – or NPV – basis over 25 years. Using NPV, an option that would involve expenditure being incurred over a long time can be compared objectively with one in which a greater proportion of the expenditure is incurred up front.' The researchers challenged the assumptions behind the cost projections used in the net present value calculation, showing that some cost items had not been treated on a comparable basis in the relative NPV evaluations. Additionally, they showed that NPV is unable to accommodate broader social costs, such as programmes of prisoner reform and rehabilitation, which are difficult to quantify.

The paper shows that questions have to be asked about the amount and timing of cash flows used in NPV calculations. Questions also have to be asked about the wider social costs and benefits that cannot be incorporated in an NPV calculation.

11.7.3 Is there some merit in the accounting rate of return?

It can be shown that there are close links between the accounting rate of return and the internal rate of return when 'economic' depreciation is applied (Stark 2004). The economic depreciation uses present value calculations that are quite complex so it is unlikely that these would be found in practice. Nevertheless, even where the accounting rate of return is different from the internal rate of return because of the depreciation method used, there may be links in terms of relative ranking of projects. The accounting rate of return may be regarded as an estimate which has some relationship with the internal rate of return, providing the effect of depreciation is understood. Comparisons within industries might have some meaning, where depreciation policies are similar.

11.8 Summary

Key themes in this chapter are:

- **Capital investment appraisal** is the application of a set of methods of quantitative analysis which give guidance to managers in making decisions as to how best to invest long-term funds. Four methods of quantitative analysis are explained in the chapter.
- The **payback period** is the length of time required for a stream of net cash inflows from a project to equal the original cash outlay.
- The **accounting rate of return** is calculated by taking the average annual profits expected from a project as a percentage of the capital invested.
- The process of calculating present value is called **discounting**. The interest rate used is called the **discount rate**. The net present value method of investment appraisal and the internal rate of return method are both based on discounting.
- The **net present value** of a project is equal to the present value of the cash inflows minus the present value of the cash outflows, all discounted at the cost of capital. The decision rules are:
  - Where the net present value of the project is *positive*, accept the project.
  - Where the net present value of the project is *negative*, reject the project.
  - Where the net present value of the project is zero, the project is acceptable in meeting the cost of capital but gives no surplus to its owners.

- The **internal rate of return** (IRR) is the discount rate at which the present value of the cash flows generated by the project is equal to the present value of the capital invested, so that the net present value of the project is zero. The decision rules are:
  - Where the IRR of the project is greater than the cost of capital, accept the project.
  - Where the IRR of the project is less than the cost of capital, reject the project.
  - Where the IRR of the project equals the cost of capital, the project is acceptable in meeting the required rate of return of those investing in the business but gives no surplus to its owners.

References and further reading

Brounen, D., de Jong, A. and Koedijk, K. (2004) 'Corporate finance in Europe: confronting theory with practice', *Financial Management*, Tampa, USA, 33(4): 71–101.

Cooper, C. and Taylor, P. (2005) 'Independently verified reductionism: prison privatisation in Scotland', *Human Relations*, April, 58: 497–522.

Stark, A. (2004) 'Estimating economic performance from accounting data: a review and synthesis', *The British Accounting Review*, 36: 321–43.

QUESTIONS

The Questions section of each chapter has three types of question. '**Test your understanding**' questions to help you review your reading are in the 'A' series of questions. You will find the answer to these by reading and thinking about the material in the textbook. '**Application**' questions to test your ability to apply technical skills are in the 'B' series of questions. Questions requiring you to show skills in '**Problem solving and evaluation**' are in the 'C' series of questions. The symbol **[S]** indicates that a solution is available at the end of the book.

A Test your understanding

A11.1 What is the purpose of capital investment appraisal (section 11.1)?

A11.2 What is meant by the assumption of certainty of cash flows (section 11.1.2)?

A11.3 What are the main steps in making a decision about a capital investment (section 11.1.3)?

A11.4 What is the payback method of evaluating a project (section 11.2)?

A11.5 What are the advantages and limitations of the payback method (section 11.2.3)?

A11.6 What is the accounting rate of return (section 11.3)?

A11.7 What are the advantages and limitations of the accounting rate of return as a technique for use in capital investment appraisal (section 11.3.2)?

A11.8 What is meant by the time value of money (section 11.4.1)?

A11.9 What is meant by the present value of a cash flow (section 11.4.1)?

A11.10 What is meant by the term 'discounting' (section 11.4.1)?

A11.11 Define net present value and explain how it is calculated (section 11.4.2).

A11.12 State the net present value decision rule to be used in capital investment appraisal (section 11.4.2).

A11.13 How is the cost of capital decided upon (section 11.4.3)?

A11.14 Define internal rate of return and explain how it is calculated (section 11.5).

A11.15 State the internal rate of return decision rule to be used in capital investment appraisal (section 11.5.2).

A11.16 **[S]** Calculate the present value of £100 receivable at the end of (a) one year, (b) two years and (c) three years, using a discount rate of 8 per cent per annum.

A11.17 **[S]** Calculate the present value of £100 receivable at the end of five years using a discount rate of (a) 4 per cent, (b) 6 per cent and (c) 8 per cent per annum.

B Application

B11.1 [S]

Projects Ltd intends to acquire a new machine costing £50,000 which is expected to have a life of five years, with a scrap value of £10,000 at the end of that time.

Cash flows arising from operation of the machine are expected to arise on the last day of each year as follows:

End of year	£
1	10,000
2	15,000
3	20,000
4	25,000
5	25,000

Calculate the payback period, the accounting rate of return and the net present value, explaining the meaning of each answer you produce. (Assume a discount rate of 10 per cent per annum.)

B11.2 [S]

In a calculation of the internal rate of return of a project it is found that the net present value is +£122m at 22 per cent discount rate and –£58m at 24 per cent discount rate. What is the Internal Rate of Return?

B11.3 [S]

XYZ Ltd is considering purchasing a new machine, and the relevant facts concerning two possible choices are as follows:

	Machine A	*Machine B*
Capital expenditure required	£65,000	£60,000
Estimated life in years	4	4
Residual value	nil	nil
Cash flow after taxation each year	£25,000	£24,000

The company's cost of capital is 10 per cent.

Required

Calculate, for each machine, the payback period, the net present value and the profitability index. State, with reasons, which machine you would recommend.

B11.4

In a calculation of the internal rate of return of a project it is found that the net present value is +£60m at 24 per cent discount rate and –£20m at 26 per cent discount rate. What is the Internal Rate of Return?

B11.5 [S] [CIMA question]
A company is considering investing $100,000 in a new machine that will reduce its annual cash operating costs as follows:

Year	*Operating cash costs saved* $000
1	35
2	45
3	55
4	30

Calculate the payback period to the nearest 0.1 years.

CIMA Paper P2 – Management Accounting – Decision Management November 2008, Question 1.5

C Problem solving and evaluation

C11.1 [S]
Marsh Limited has investigated the possibility of investing in a new machine. The following data have been extracted from the report relating to the project:

Cost of machine on 1 January Year 6: £500,000.

Estimated scrap value at end of Year 5: Nil

Year	*Net cash flows* £000
1	50
2	200
3	225
4	225
5	100

The company's cost of capital is 8%.

Required
Evaluate the acceptability of the project using the net present value method of investment appraisal.

C11.2
BY Ltd is considering carrying out a major programme of staff training. The training scheme will cost £100,000 and will be paid for immediately. It is expected to produce additional cash flows as follows:

	Additional cash inflow
One year from today	£50,000
Two years from today	£40,000
Three years from today	£30,000
Four years from today	£30,000
Five years from today	£20,000

The cost of capital to the company is 6 per cent.

Required
(a) Evaluate the expenditure on the staff training scheme.
(b) Comment on other factors to be considered before taking up the training scheme.

C11.3
HOP Ltd forecasts cash flows of £30,000 per annum for four years. It will invest £80,000 in fixed assets having a four-year life and no residual value.

Calculate: (a) the accounting rate of return and (b) the internal rate of return.

Case studies

Real world cases

Prepare short answers to Case studies **11.1**, **11.2** and **11.3**.

Case 11.4

Using a suitable computer spreadsheet package, set up a spreadsheet which will calculate net present values and internal rates of return for projects having cash flows for a ten-year period. Test the spreadsheet with sample data and then write a brief instruction sheet. Save the spreadsheet to a disk and exchange disks and instruction sheets with another group in the class.

Case 11.5

Now exchange your spreadsheet with that of another student and write an evaluation of the spreadsheet you have received from the other person. Consider the following:

(a) Does it deal with all possible types of cash flows (e.g. a negative flow at some point)?
(b) Does it provide a recommendation on accept/reject (e.g. using a conditional function)?
(c) Does it allow for relatively easy variation of the discount rate?
(e) Does the instruction sheet explain how to produce graphs of net present value plotted against discount rate?

List any other features of the spreadsheet which you would use in evaluating its effectiveness and user-friendliness.

Appendix: table of discount factors

Present value of £1 to be received after *n* years when the rate of interest is *r*% per annum equals $1/(1 + r)^n$.

Number of years	*1%*	*2%*	*3%*	*4%*	*5%*	*6%*	*7%*	*8%*	*9%*	*10%*	*11%*	*12%*	*13%*	*14%*	*15%*
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	0.901	0.893	0.885	0.877	0.870
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826	0.812	0.797	0.783	0.769	0.756
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751	0.731	0.712	0.693	0.675	0.658
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683	0.659	0.636	0.613	0.592	0.572
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621	0.593	0.567	0.543	0.519	0.497
6	0.942	0.888	0.837	0.790	0.746	0.705	0.666	0.630	0.596	0.564	0.535	0.507	0.480	0.456	0.432
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513	0.482	0.452	0.425	0.400	0.376
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467	0.434	0.404	0.376	0.351	0.327
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424	0.391	0.361	0.333	0.308	0.284
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386	0.352	0.322	0.295	0.270	0.247
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350	0.317	0.287	0.261	0.237	0.215
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319	0.286	0.257	0.231	0.208	0.187
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290	0.258	0.229	0.204	0.182	0.163
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263	0.232	0.205	0.181	0.160	0.141
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239	0.209	0.183	0.160	0.140	0.123
16	0.853	0.728	0.623	0.534	0.458	0.394	0.339	0.292	0.252	0.218	0.188	0.163	0.141	0.123	0.107
17	0.844	0.714	0.605	0.513	0.436	0.371	0.317	0.270	0.231	0.198	0.170	0.146	0.125	0.108	0.093
18	0.836	0.700	0.587	0.494	0.416	0.350	0.296	0.250	0.212	0.180	0.153	0.130	0.111	0.095	0.081
19	0.828	0.686	0.570	0.475	0.396	0.331	0.277	0.232	0.194	0.164	0.138	0.116	0.098	0.083	0.070
20	0.820	0.673	0.554	0.456	0.377	0.312	0.258	0.215	0.178	0.149	0.124	0.104	0.087	0.073	0.061
21	0.811	0.660	0.538	0.439	0.359	0.294	0.242	0.199	0.164	0.135	0.112	0.093	0.077	0.064	0.053
22	0.803	0.647	0.522	0.422	0.342	0.278	0.226	0.184	0.150	0.123	0.101	0.083	0.068	0.056	0.046
23	0.795	0.634	0.507	0.406	0.326	0.262	0.211	0.170	0.138	0.112	0.091	0.074	0.060	0.049	0.040
24	0.788	0.622	0.492	0.390	0.310	0.247	0.197	0.158	0.126	0.102	0.082	0.066	0.053	0.043	0.035
25	0.780	0.610	0.478	0.375	0.295	0.233	0.184	0.146	0.116	0.092	0.074	0.059	0.047	0.038	0.030
26	0.772	0.598	0.464	0.361	0.281	0.220	0.172	0.135	0.106	0.084	0.066	0.053	0.042	0.033	0.026
27	0.764	0.586	0.450	0.347	0.268	0.207	0.161	0.125	0.098	0.076	0.060	0.047	0.037	0.029	0.023
28	0.757	0.574	0.437	0.333	0.255	0.196	0.150	0.116	0.090	0.069	0.054	0.042	0.033	0.026	0.020
29	0.749	0.563	0.424	0.321	0.243	0.185	0.141	0.107	0.082	0.063	0.048	0.037	0.029	0.022	0.017
30	0.742	0.552	0.412	0.308	0.231	0.174	0.131	0.099	0.075	0.057	0.044	0.033	0.026	0.020	0.015

continued

Number of years	*16%*	*17%*	*18%*	*19%*	*20%*	*21%*	*22%*	*23%*	*24%*	*25%*	*26%*	*27%*	*28%*	*29%*	*30%*
1	0.862	0.855	0.847	0.840	0.833	0.826	0.820	0.813	0.806	0.800	0.794	0.787	0.781	0.775	0.769
2	0.743	0.731	0.718	0.706	0.694	0.683	0.672	0.661	0.650	0.640	0.630	0.620	0.610	0.601	0.592
3	0.641	0.624	0.609	0.593	0.579	0.564	0.551	0.537	0.524	0.512	0.500	0.488	0.477	0.466	0.455
4	0.552	0.534	0.516	0.499	0.482	0.467	0.451	0.437	0.423	0.410	0.397	0.384	0.373	0.361	0.350
5	0.476	0.456	0.437	0.419	0.402	0.386	0.370	0.355	0.341	0.328	0.315	0.303	0.291	0.280	0.269
6	0.410	0.390	0.370	0.352	0.335	0.319	0.303	0.289	0.275	0.262	0.250	0.238	0.227	0.217	0.207
7	0.354	0.333	0.314	0.296	0.279	0.263	0.249	0.235	0.222	0.210	0.198	0.188	0.178	0.168	0.159
8	0.305	0.285	0.266	0.249	0.233	0.218	0.204	0.191	0.179	0.168	0.157	0.148	0.139	0.130	0.123
9	0.263	0.243	0.225	0.209	0.194	0.180	0.167	0.155	0.144	0.134	0.125	0.116	0.108	0.101	0.094
10	0.227	0.208	0.191	0.176	0.162	0.149	0.137	0.126	0.116	0.107	0.099	0.092	0.085	0.078	0.073
11	0.195	0.178	0.162	0.148	0.135	0.123	0.112	0.103	0.094	0.086	0.079	0.072	0.066	0.061	0.056
12	0.168	0.152	0.137	0.124	0.112	0.102	0.092	0.083	0.076	0.069	0.062	0.057	0.052	0.047	0.043
13	0.145	0.130	0.116	0.104	0.093	0.084	0.075	0.068	0.061	0.055	0.050	0.045	0.040	0.037	0.033
14	0.125	0.111	0.099	0.088	0.078	0.069	0.062	0.055	0.049	0.044	0.039	0.035	0.032	0.028	0.025
15	0.108	0.095	0.084	0.074	0.065	0.057	0.051	0.045	0.040	0.035	0.031	0.028	0.025	0.022	0.020
16	0.093	0.081	0.071	0.062	0.054	0.047	0.042	0.036	0.032	0.028	0.025	0.022	0.019	0.017	0.015
17	0.080	0.069	0.060	0.052	0.045	0.039	0.034	0.030	0.026	0.023	0.020	0.017	0.015	0.013	0.012
18	0.069	0.059	0.051	0.044	0.038	0.032	0.028	0.024	0.021	0.018	0.016	0.014	0.012	0.010	0.009
19	0.060	0.051	0.043	0.037	0.031	0.027	0.023	0.020	0.017	0.014	0.012	0.011	0.009	0.008	0.007
20	0.051	0.043	0.037	0.031	0.026	0.022	0.019	0.016	0.014	0.012	0.010	0.008	0.007	0.006	0.005
21	0.044	0.037	0.031	0.026	0.022	0.018	0.015	0.013	0.011	0.009	0.008	0.007	0.006	0.005	0.004
22	0.038	0.032	0.026	0.022	0.018	0.015	0.013	0.011	0.009	0.007	0.006	0.005	0.004	0.004	0.003
23	0.033	0.027	0.022	0.018	0.015	0.012	0.010	0.009	0.007	0.006	0.005	0.004	0.003	0.003	0.002
24	0.028	0.023	0.019	0.015	0.013	0.010	0.008	0.007	0.006	0.005	0.004	0.003	0.003	0.002	0.002
25	0.024	0.020	0.016	0.013	0.010	0.009	0.007	0.006	0.005	0.004	0.003	0.003	0.002	0.002	0.001
26	0.021	0.017	0.014	0.011	0.009	0.007	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001
27	0.018	0.014	0.011	0.009	0.007	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001
28	0.016	0.012	0.010	0.008	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001
29	0.014	0.001	0.008	0.006	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.000
30	0.012	0.009	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.001	0.001	0.001	0.001	0.000	0.000